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THE CONTENT OF A MATHEMATICAL COURSE FOR THE JUNIOR HIGH SCHOOL.*

BY F. W. GENTLEMAN.

In view of the fact that junior high schools are actually being established in different parts of New England, it becomes the duty of this association to consider what shall be the nature of the work in mathematics for the course.

The junior-high-school period, in general, comprises the seventh, eighth and ninth school years, so the outline I am to present will be for a three-year course. One of the changes to be made for junior high schools is the gradual introduction of departmental work to bridge the gap from one-teacher to many-teacher instruction. This presupposes well-prepared teachers. Another change is the unification of the work in each subject, resulting in the establishment of a more connected, more logical, system. Hence it will be possible to offer much earlier some of the less difficult and more useful of the present high-school material, and to defer some of the more difficult and less useful material now offered in the elementary schools. If we attempt to saw off a strip of the present high-school course and nail it to a strip sawed off from the present elementary-school course and to claim thereby to have made a junior-high-school course, we are surely deceiving ourselves and defeating one of the main purposes, I believe, for which the junior high schools are being established. The change offers an opportunity for very necessary reforms in the content of the course and in the method of presentation.

Instruction in mathematics in the junior high school must necessarily begin where the pupil has reached as a result of the work in arithmetic for the first six years. It is commonly conceded that during this school period (Grades I.–VI.) he should have mastered the mechanics of arithmetic, and that the six years of school time is ample for the accomplishment of this purpose. By the mechanics of arithmetic I mean the processes of addition, subtraction, multiplication and division of integers, common fractions and decimal fractions. His attention thus far has been necessarily focused on the individual figures of numbers rather than on the number values so expressed.

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* Read at the Springfield meeting of the Association of Teachers of Mathematics in New England, March 3, 1917.

In the mathematical work of the junior high school, the pupil should accustom himself to the standards of the business world; namely, that an example done once without review or check is only half done and that the responsibility for the correctness of the work must rest with the computer. The first of these means that the pupil must be shown, and compelled to use systematically, some method of reviewing or checking up his work. The second means that the responsibility for the correctness of his work must fall upon the pupil himself, and not upon some authority over him.

In the operation of addition and subtraction of numbers, no result should be accepted which does not carry with it the evidence of having been checked up. An accurate check in multiplication is performed by interchanging the multiplier and the multiplicand. An accurate check in division is performed by multiplying the quotient by the divisor and adding the remainder.

In the operation of multiplication and division, results should be estimated before any computations are performed. These estimates should be part and parcel of all the work submitted. This means that emphasis should be placed upon rational or common sense methods of locating the decimal point. Furthermore, if the method of multiplication taught were one in which the figures of the multiplier were used from left to right, then the approximation and the mechanics would go hand in hand; since the decimal point is located in the first partial product and the succeeding partial products are added as corrections thereto. This method, in later scientific work where approximate numbers are involved, lends itself to a considerable economy in the number of figures used.

In all this work, and in that which follows, process and speed should not be stressed to the sacrifice of judgment regarding results, nor to the sacrifice of the accuracy of results. Again, the choice of problems for application here and later should be more carefully considered than appears to have been done in many current textbooks. Problems coming from impossible conditions, and naming impossible prices, are to be avoided, as well as problems that are mere collections of words to make some process necessary. Advertised sales, household budgets, statistical reports, etc., offer legitimate sources for problems. Furthermore, if we are to ground the pupil thoroughly in the fundamentals of arithmetic, so that the business world will be better satisfied with our product, we should make the wording of the problem simple and direct until the one principle involved in that problem has been mastered by the pupil. One of the main weaknesses in our problem work of to-day in the elementary course, is that many problems are so involved that the pupil becomes accustomed to failure and loses confidence in his ability to handle any arithmetical computations that require thought on his part.

Concerning the subject of percentage, from my teaching experience I am convinced that the presentation of its elements in the usual order of Cases I. and II. should be reversed. That is to say, that first the pupil should learn to interpret and to express, as per cent. relations, those relations between the number data in the fields familiar to his common experience. It is necessary for him to get a clear conception of the idea of per cent. as so many hundredths, or such a fractional part of a given amount. After the pupil has comprehended the per hundred idea and has realized that the per cent. obtained is a per cent. of some definite amount, then he may use intelligently the per centum idea in finding per cents of amounts; he is no longer in the dark as to the interpretation of the computation called for in these problems, so in making his computations he can proceed to apply common sense.

In considering the applications of percentage, it is important to note that the per centum idea has a much wider field of application than the monetary field alone. These applications should include problems concerning school data, comparisons of lines and of areas, and comparisons from the field of statistics, etc.

The third case, so-called, of percentage, the formal methods of computing interest, successive discounts, marking goods to sell at a gain per cent. on the marked price, the computing of per cents in statistical work to certain degrees of accuracy, introductory treatment of taxes and insurance, should be deferred until the second year.

The equation should claim an early place in the course of the junior high school. It naturally occurs in the following topics: statements of elementary number-facts; formulas of mensuration, about which I shall speak at length later; statement of the equality of two ratios, here serving as a simple means of approach to the ratio idea involved in the per cent. relation; formulas for percentage, for interest and for scientific facts; and statement of general problems not included in any of the above, many of which problems have heretofore required a solution by the analytic method. The equation affords a simple direct method of expressing mathematical relations. By its use mathematical solutions are clarified. It is needed by the future mechanic and other tradesmen if they are to read trade journals intelligently, since the equation is the world's way of expressing a rule.

The kind of equation most needed by the pupil in the first and second year of the junior high school, is that which requires the axioms of multiplication and division for its solution. The treatment of the equation in these years should be natural and informal. The pupil should indicate definitely his progress, step by step, and should definitely assure himself of the correctness of his result by a check.

Another topic that should occupy an important place in the early part of the junior-high-school course is the measurement of the familiar geometrical figures and the drawing of these figures to full size and to scale. Familiarity with their shapes and properties, and a knowledge of the terms applied to them will remove many of the difficulties and misconceptions that are met later in the systematic study of geometry. For this work in measurement, the pupil should be supplied with a protractor for angle measurement, and with a ruler on which the inch is graduated to tenths as well as to sixteenths and the foot graduated to hundredths. With such a ruler measurements may be obtained in decimal fractions as well as common fractions, thereby extending the work in decimal computation beyond the monetary field. Such a ruler would be especially useful in making scale drawings.

Special emphasis should be placed upon the care with which the measurements are made. Here may be made a study of the shape and properties of the square, rectangle, triangle, parallelogram and trapezoid. For the second year, this work should be extended to include the use of the compass for simple geometrical constructions, such as the drawing of perpendiculars, parallels, bisectors, etc. At this time the different plane figures should be systematically grouped and their common properties studied.

The graphical representation of number-data from the field of statistics has a place in the junior-high-school course. By comparing the lengths of lines and rectangles and parts of circles representing certain groups of facts, the pupil may learn to read number-data so expressed and also have experience in making graphs from given data. This work will assist him in reading intelligently the many articles appearing in magazines devoted to scientific and social problems. Furthermore, through such work he can visualize the idea of "round" number, one kind of approximate number.

The mensuration of the areas of the square, the rectangle, the parallelogram, the triangle, the trapezoid, and the circle is work within the comprehension of the first-year pupil. The rules for these should be developed from diagrams and these rules expressed as formulas at once. All this work should be on a rational basis, and a formula such as $A = bh$, should mean, first of all, A (the number of square units) $= bh$ square units. For this the unit of area (a square something) must be visualized. We should be sure that the pupil carries away no such incorrect ideas as 5 in. × 8 in. = 40 sq. in. For this work in mensuration the pupil should obtain as much of his data as possible by actual measurements, then his common sense and judgment may be better trained in the use of data that he may have in any problem. If this sort of training in computation be given him in the first year of the junior high school, then in the mensuration of the second year the distinction between a number obtained by count and one obtained by measurement may be made. On this foundation, approximate computations may be taken up and the results retained to that number of figures that corresponds to the given data of the problem.

Accuracy (so-called) where results from measured data are required to several decimal places, gives the pupil the wrong idea of what is meant by mathematical accuracy. The pupil should be trained to realize what sort of an answer he ought to get, then make an effort to get the result correct to a degree that his common sense demands. When asked your age you do not say that you are 25 years, 3 months, 5 days, 4 hours, 8 minutes and 30 seconds old. Even were such an answer correct, it would be practically senseless. When such a kind of "accuracy" is required in answers, problems cease to be real and results to be of value.

This discussion concerning the rational kind of answer leads to the topic of square root, which should come in the second year. The one really rational method for getting square root is that known by some as Newton's method. The "completing the square" method quickly degenerates into the following of a mechanical rule. For Newton's method the knowledge of many squares becomes essential, the rational estimate is all-important, and the idea that the square root is one of the two equal factors of a number can not be lost sight of.

This is the natural place in the course to apply one of the most useful facts of geometry, the Pythagorean theorem. Following this, a study should be made of the development and use of the formulas for finding the surfaces and volumes of the block, the cube, the prism, the cylinder, the pyramid, and the cone. Here, too, some of the formulas based upon the simpler facts of elementary science might be introduced to advantage, as elementary science is likely to have a place in the course of the junior high school.

Near the end of the second year, equations involving the axioms for addition and subtraction should be studied, and this followed by a study of the solution of simultaneous linear equations by the method of elimination by addition or subtraction.

Throughout the course, the teacher should require that the work be neatly done; that clear, concise statements be made showing the progress of the work, when necessary; and that the computations be systematically arranged.

At the end of the first two years of the junior high school, the pupil, who has followed the course as outlined above, should be well grounded in the fundamentals of arithmetic, so that he can attack with confidence the problems that he is likely to meet, should he be forced to leave at the end of his eighth year. He should have a fairly clear idea of the shapes and properties of the common plane figures and solids. He should have some grasp of generalized arithmetic, fitting him to continue his work in his ninth year much more intelligently than he does at present.

In the last year of his junior-high-school course, he should again make a study of arithmetic, to get some idea of its unity and its general values; he should consider more extensively the applications of percentage to the field of science, getting some definite idea of the per cent. of error in data and in result, together with a more definite knowledge of the meaning of significant figures. He should make a somewhat more intensive study than before of such applications of percentage as taxes, bonds, mortgages, insurance, etc.

Once more he should deal with formulas, but this time with the transformation of those formulas already familiar to him, and with the building of new formulas from rules, whether these rules are the results of his own experience, or not; in other words, he should now be able to symbolize his scientific language.

The addition, subtraction, multiplication and division of polynomials should be studied as an aid in solving certain types of equations. A study should now be made of linear equations, involving the treatment of negative numbers; of simultaneous linear equations, including the graphical solution; of complete quadratic equations solved by factoring; and of simultaneous equations, one quadratic and one linear, solved by substitution and by graphs. Enough factoring would be necessary to make it possible to solve any quadratic having rational roots.

Near the end of this year, I would give the pupil an introduction into the systematic study of geometry, proving informally with him certain fundamental statements, and having him prove certain other statements formally, choosing for such formal demonstrations those statements the proofs for which are clear-cut and definite.

In conclusion, it seems to me that, if the course for the junior high school is arranged somewhat as I have outlined, and certain changes of method are made as outlined, then such results as the following might be expected:

A better understanding of quantitative relations;

A more common sense viewpoint concerning the value of results, with a growing respect for sensible accuracy;

More strongly developed habits of self-reliance;

A more thorough grounding in the fundamentals of arithmetical computation;

Earlier development of the power of independent reasoning in mathematical work;

For the pupil who leaves at the end of the eighth or ninth year, a greater ability to handle a variety of the mathematical tools used in the solution of the problems of everyday life;

For those who go to the senior high school, a course better fitted to suit their capacities and tendencies, since the teacher in charge would have wider opportunity than now to see the pupil's interest in, aptitude for, and capacity to grasp the mathematical point of view;

Finally, a stimulation of his interest in the further study of mathematics by giving him a clearer idea of what the subject is about, and by presenting to him a vision of the extent of the applications of mathematics to the different fields of the world's work;

For the pupil who enters the senior high school, less danger of unwise choice of mathematical work, because he already has a fair knowledge of his mathematical limitations;

Finally, a keener interest in the further study of mathematics, because he has a clearer idea of its meaning and a vision of its manifold applications to the world's important work.

PROPOSED OUTLINE.

First Year.

1. Review of fundamental operations of arithmetic.
2. Equations: (*a*) $bx=c$; (*b*) $\frac{x}{a}=\frac{b}{c}$ (ratio).
3. Measurement of straight lines, angles and plane rectilinear figures. Drawing to scale. Straight line graphs.

4. Percentage: (*a*) Finding what part one number is of another.
    (*b*) Finding percents of given amounts.
    (*c*) Applications: single discount, simple interest, commission.
5. Mensuration: (*a*) Areas of rectangular figures and the circle.
    (*b*) Rectangular and circular graphs.

6 Summary: Applied problems.

Second Year.

1. Review of fundamental operations of arithmetic applied to business transactions; approximate products.
2. Formulas: (*a*) Perimeters and areas of plane figures.
    (*b*) Square root; Pythagorean theorem.
    (*c*) Surfaces and volumes of solids.
3. Percentage: (*a*) Finding the base, percentage and rate given.
    (*b*) Applications: successive discounts, interest (formal method), notes, savings banks, taxes, insurance.
4. Construction of geometrical figures; classification of plane figures; graphical representation of statistics by broken line.
5. Equations: (*a*) $ax + b = cx + d$.
    (*b*) Simultaneous linear.

Third Year.

1. Review of arithmetic, to include bonds, mortgages, taxes, insurance, significant figures.
2. Formulas: (*a*) Transformation of formulas.
    (*b*) Construction of formulas.
3. Linear equations: (*a*) Involving addition and subtraction of polynomials.
    (*b*) Involving multiplication and division of polynomials.
    (*c*) Involving negative numbers.
    (*d*) Simultaneous (graphs).
4. Factoring: (*a*) $aQ + bQ - cQ$; (*b*) $Q^2 - O^2$; (*c*) $Q^2 + 2QO + O^2$; (*d*) $Q^2 + aQ + b$; (*e*) $aQ^2 + bQ + c$.

5. Quadratic equations: (*a*) Pure; (*b*) complete (solved by factoring); (*c*) simultaneous, one quadratic and one linear (solved by substitution and by graphs).
6. Introduction to systematic study of theorems of geometry.

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